

Generation of longitudinal electric current by transversal electromagnetic field in Maxwellian plasmas

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Abstract

The analysis of nonlinear interaction of transversal electromagnetic field with Maxwellian collisionless classical and quantum plasmas is carried out. Formulas for calculation electric current in Maxwellian collisionless classical and quantum plasmas are deduced. It has appeared, that the nonlinearity account leads to occurrence of the longitudinal electric current directed along a wave vector. This second current is orthogonal to the known transversal current, received at the classical linear analysis. Graphic comparison of density of electric current for classical Maxwellian plasmas and Fermi–Dirac plasmas (plasmas with any degree of degeneration of electronic gas) is carried out. Graphic comparison of density of electric current for classical and quantum Maxwellian plasmas is carried out. Also comparison of dependence of density of electric current of quantum Maxwellian plasmas from dimensionless wave number at various values of dimensionless frequency of oscillations of electromagnetic field is carried out.

Key words: collisionless plasmas, Vlasov equation, Maxwellian plasma, Wigner integral, quantum distribution function, longitudinal electrical current.

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Introduction

The nonlinear phenomena in plasma are studied during the long time [1] – [7]. In work [8] was the approach to studying of dielectric function

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in quantum plasma is offered. Then in our works [9]–[12] was dielectric permeability in the quantum collisional to plasma is investigated.

In works [13] and [14] plasma with any degeneration of electronic gas (plasma of Fermi–Dirac) was considered. It has been shown, that with use square-law decomposition of function of distribution is possible to reveal longitudinal electric current. This current is generated by the transversal electromagnetic field. Longitudinal current is perpendicular to the transversal current which is at the linear analysis.

Let us notice, that existence of the longitudinal current generated by transversal electromagnetic field, it has been noticed more half century ago in work [2].

In the present work formulas for calculation electric current into Maxwellian collisionless plasma at any temperature (at any degrees of degeneration of the electronic gas) are deduced.

It has appeared, that electric current expression consists of two summands. The first summand, linear on vector potential, is known classical expression of electric current. This electric current is directed along vector potential of electromagnetic field. The second summand represents itself electric current, which is proportional to the square vector potential of electromagnetic field. The second current is perpendicular to the first and it is directed along the wave vector. Occurrence of the second current comes to light the spent account nonlinear character interactions of electromagnetic field with plasma.

Then the kinetic equation with Wigner integral concerning quantum function of distribution is used. The nonlinear analysis is made and the formula for calculation longitudinal electric current is deduced. This current also is generated by transversal electromagnetic field.

1. Vlasov kinetic equation and its solution

Let us consider Vlasov equation describing behaviour of collisionless

plasmas

$$\frac{\partial f}{\partial t} + \mathbf{v} \frac{\partial f}{\partial \mathbf{r}} + e \left(\mathbf{E} + \frac{1}{c} [\mathbf{v}, \mathbf{H}] \right) \frac{\partial f}{\partial \mathbf{p}} = 0. \quad (1.1)$$

Vector potential we take as orthogonal to direction of a wave vector \mathbf{k}

$$\mathbf{k} \mathbf{A}(\mathbf{r}, t) = 0. \quad (1.2)$$

in the form of the running harmonious wave

$$\mathbf{A}(\mathbf{r}, t) = \mathbf{A}_0 e^{i(\mathbf{k}\mathbf{r} - \omega t)}.$$

Scalar potential we will consider equal to zero. Electric and magnetic fields are connected with vector potential by equalities

$$\mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{A}}{\partial t}, \quad \mathbf{H} = \text{rot} \mathbf{A}. \quad (1.3)$$

The wave vector we direct along axis x : $\mathbf{k} = k(1, 0, 0)$, and vector potential of electromagnetic field we direct along axis y

$$\mathbf{A} = A_y(x, t)(0, 1, 0), \quad A_y(x, t) \sim e^{i(kx - \omega t)}.$$

Then

$$A_y = -\frac{ic}{\omega} E_y, \quad \mathbf{H} = \frac{ck}{\omega} E_y(0, 0, 1),$$

$$[\mathbf{v}, \mathbf{H}] = \frac{ck}{\omega} E_y(v_y, -v_x, 0).$$

Let us operate with method of consecutive approximations. Considering, that the member with an electromagnetic field has an order, on unit smaller other members, let us rewrite the equation (1.1) in the form

$$\begin{aligned} & \frac{\partial f^{(k)}}{\partial t} + v_x \frac{\partial f^{(k)}}{\partial x} + \\ & + e E_y \left(\frac{\partial f^{(k-1)}}{\partial p_y} \left(1 - \frac{kv_x}{\omega} \right) + \frac{kv_y}{\omega} \frac{\partial f^{(k-1)}}{\partial p_x} \right) = 0, \quad k = 1, 2. \end{aligned} \quad (1.4)$$

Here in zero approximation $f^{(0)}$ is the absolute Maxwell—Boltzmann distribution,

$$f^{(0)} = f_0 = N \left(\frac{\beta}{\pi} \right)^{3/2} e^{-\beta v^2}, \quad \beta = \frac{m}{2k_B T},$$

k_B is the Boltzmann constant, T is the plasmas temperature.

It is easy to see, that $\mathbf{P} = \frac{\mathbf{v}}{v_T} = \frac{\mathbf{p}}{p_T}$ is the dimensionless electron velocity (or momentum), v_T is the thermal velocity of electrons,

$$v_T = \sqrt{\frac{2k_B T}{m}}.$$

We notice that

$$[\mathbf{v}, \mathbf{H}] \frac{\partial f^{(0)}}{\partial \mathbf{p}} = 0,$$

because

$$\frac{\partial f^{(0)}}{\partial \mathbf{p}} \sim \mathbf{v}.$$

Therefore in first approximation Vlasov equation have the form

$$\frac{\partial f^{(1)}}{\partial t} + v_x \frac{\partial f^{(1)}}{\partial x} = -eE_y \frac{\partial f^{(0)}}{\partial p_y}. \quad (1.5)$$

And in second approximation Vlasov equation have the following form

$$\begin{aligned} & \frac{\partial f^{(2)}}{\partial t} + v_x \frac{\partial f^{(2)}}{\partial x} = \\ & = -eE_y \left(\frac{\partial f^{(1)}}{\partial p_y} \cdot \frac{\omega - kv_x}{\omega} + \frac{kv_y}{\omega} \frac{\partial f^{(1)}}{\partial p_x} \right). \end{aligned} \quad (1.6)$$

We search solution in first approximation in the form

$$f^{(1)} = f^{(0)} + f_1(x, t, P_x), \quad f_1 \sim E_y(x, t).$$

In this approximation the equation (1.5) becomes simpler

$$\frac{\partial f_1}{\partial t} + v_T P_x \frac{\partial f_1}{\partial x} = -\frac{eE_y}{mv_T} \frac{\partial f_0}{\partial P_y}.$$

From this equation we find

$$-i(\omega - kv_T P_x) f_1 = \frac{2eE_y}{p_T} f_0(P) P_y,$$

from which

$$f_1 = \frac{2ieE_y}{p_T} \frac{P_y f_0(P)}{\omega - kv_T P_x}. \quad (1.7)$$

In the second approximation for function $f^{(2)}$ we search in the form

$$f^{(2)} = f^{(1)} + f_2(x, t, v_x), \quad f_2 \sim E_y^2(x, t).$$

Let us substitute $f^{(2)}$ in (1.6). We receive the equation

$$\frac{\partial f_2}{\partial t} + v_x \frac{\partial f_2}{\partial x} = -\frac{eE_y}{\omega} \left[(\omega - kv_x) \frac{\partial f_1}{\partial p_y} + kv_y \frac{\partial f_1}{\partial p_x} \right].$$

We transform this equation to dimensionless parameters

$$\frac{\partial f_2}{\partial t} + v_T P_x \frac{\partial f_2}{\partial x} = -\frac{eE_y}{\omega p_T} \left[(\omega - kv_T P_x) \frac{\partial f_1}{\partial P_y} + kv_T P_y \frac{\partial f_1}{\partial P_x} \right].$$

From this equation we find

$$f_2 = \frac{e^2 E_y^2}{\omega p_T^2 (\omega - kv_T P_x)} \left[\frac{\partial (P_y f_0(P))}{\partial P_y} + kv_T P_y^2 \frac{\partial}{\partial P_x} \left(\frac{f_0(P)}{\omega - kv_T P_x} \right) \right]. \quad (1.8)$$

We introduce new parameters q and Ω , q is the dimensionless wave number, Ω is the dimensionless frequency of electromagnetic field,

$$q = \frac{k}{k_T}, \quad \Omega = \frac{\omega}{k_T v_T}, \quad k_T = \frac{p_T}{\hbar} = \frac{mv_T}{\hbar}.$$

Then we rewrite the equation (1.8) in the form

$$f_2 = \frac{e^2 E_y^2}{p_T^2 k_T^2 v_T^2 (\Omega - q P_x)} \left[\frac{\partial (P_y f_0(P))}{\partial P_y} + q P_y^2 \frac{\partial}{\partial P_x} \left(\frac{f_0(P)}{\Omega - q P_x} \right) \right]. \quad (1.8')$$

Distribution function in square-law approximation on the field it is constructed

$$f = f^{(2)} = f^{(0)} + f_1 + f_2, \quad (1.9)$$

where f_1, f_2 are given accordingly by formulas (1.7) and (1.8).

2. Density of electric current

Let us calculate current density

$$\mathbf{j} = e \int \mathbf{v} f d^3 v = ev_T^4 \int \mathbf{P} f d^3 P = ev_T^4 \int \mathbf{P} f_1 d^3 P. \quad (2.1)$$

By means of (1.8) it is visible, that the vector of current density has two non-zero components

$$\mathbf{j} = (j_x, j_y, 0).$$

Here j_y is the density of known transversal current, calculated as

$$j_y = ev_T^4 \int P_y f d^3 P = ev_T^4 \int P_y f_1 d^3 P. \quad (2.2)$$

This current is directed along electric field, its density is deduced by means of linear approximation of distribution function.

Square-law on quantity of an electromagnetic field composed f_2 the contribution to density of a current does not bring. Density of transversal current is calculated under the formula

$$j_y = \frac{2ie^2 v_T^2}{mk_T} E_y(x, t) \int \frac{P_y^2 f_0(P) d^3 P}{\Omega - qP_x}. \quad (2.3)$$

Here k_T is the thermal wave number,

$$k_T = \frac{p_T}{\hbar} = \frac{mv_T}{\hbar}.$$

Let us calculate the longitudinal current. For density of longitudinal current according to definition it is had

$$j_x = e \int v_x f d^3 v = ev_T^4 \int P_x f_2 d^3 P.$$

Having taken advantage (1.8'), from here we receive, that

$$j_x = \frac{e^3 E_y^2}{k_T^2 m^2 \Omega} \int \left[\frac{\partial(P_y f_0(P))}{\partial P_y} + qP_y^2 \frac{\partial}{\partial P_x} \left(\frac{f_0(P)}{\Omega - qP_x} \right) \right] \frac{P_x d^3 P}{\Omega - qP_x}. \quad (2.4)$$

The first integral from (2.4) is equal to zero. Really, we will consider internal integral on P_y

$$\int_{-\infty}^{\infty} \frac{\partial}{\partial P_y} (P_y f_0(P)) dP_y = P_y f_0(P) \Big|_{P_y=-\infty}^{P_y=+\infty} = 0.$$

Now in the second integral from (2.4) we will calculate internal integral on P_x

$$\begin{aligned} & \int_{-\infty}^{\infty} \frac{\partial}{\partial P_x} \left(\frac{f_0(P)}{\Omega - qP_x} \right) \frac{P_x dP_x}{\Omega - qP_x} = \\ & = \frac{f_0(P)P_x}{(\Omega - qP_x)^2} \Big|_{P_x=-\infty}^{P_x=+\infty} - \int_{-\infty}^{\infty} \frac{f_0(P)}{\Omega - qP_x} d\left(\frac{P_x}{\Omega - qP_x} \right) = \\ & = -\Omega \int_{-\infty}^{\infty} \frac{f_0(P) dP_x}{(\Omega - qP_x)^3}. \end{aligned}$$

Thus, equality (2.5) becomes simpler

$$j_x = -\frac{e^3 E_y^2}{k_T^2 m^2} q \int \frac{f_0(P) P_y^2 d^3 P}{(\Omega - qP_x)^3}. \quad (2.5)$$

Double internal integral from (2.5) in plane (P_y, P_z) it is calculated in polar coordinates $(P_y = \rho \cos \varphi, P_z = \rho \sin \varphi)$

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-P_y^2 - P_z^2} P_y^2 dP_y dP_z = \int_0^{2\pi} \int_0^{\infty} \cos^2 \varphi e^{-\rho^2} \rho^3 d\varphi d\rho = \frac{\pi}{2}.$$

Hence, the density of longitudinal current is equal to

$$j_x = \frac{e^3 E_y^2 N q}{2k_T^2 m^2 v_T^3 \sqrt{\pi}} \int_{-\infty}^{\infty} \frac{e^{-P_x^2} dP_x}{(qP_x - \Omega)^3}. \quad (2.6)$$

Выражение (2.6) представим в виде:

$$j_x^{\text{long}} = J_c(\Omega, q) \sigma_{l,tr} k E_y^2. \quad (2.7)$$

В (2.7) $\sigma_{l,tr}$ – продольно–поперечная проводимость, $J_c(\Omega, q)$ – безразмерная часть продольного тока,

$$\sigma_{l,tr} = \frac{e}{p_T k_T} \Omega_p^2 = \frac{e\hbar}{p_T^2} \left(\frac{\hbar \omega_p}{m v_T^2} \right)^2 = \frac{e\hbar}{p_T^2} \left(\frac{\omega_p}{k_T v_T} \right)^2,$$

$$J_c(\Omega, q) = \frac{1}{8\pi\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{e^{-P_x^2} dP_x}{(qP_x - \Omega)^3},$$

ω_p – плазменная (лэнгмюровская) частота, Ω_p – безразмерная плазменная частота,

$$\omega_p = \sqrt{\frac{4\pi e^2 N}{m}}, \quad \Omega_p = \frac{\omega_p}{k_T v_T}.$$

Let us present the formula (2.6) in an invariant form. For this purpose we will introduce transversal electric field

$$\mathbf{E}_{\text{tr}} = \mathbf{E} - \frac{\mathbf{k}(\mathbf{E}\mathbf{k})}{k^2} = \mathbf{E} - \frac{\mathbf{q}(\mathbf{E}\mathbf{q})}{q^2}.$$

Now equality (2.7) we will present in coordinate-free form

$$\mathbf{j}^{\text{long}} = J_c(\Omega, q) \sigma_{l, \text{tr}} \mathbf{k} \mathbf{E}_{\text{tr}}^2 = J_c(\Omega, q) \sigma_{l, \text{tr}} \frac{\omega}{c} [\mathbf{E}, \mathbf{H}].$$

The integral (2.6) is calculated with use of known rule of Landau as Cauchy type integral

$$\begin{aligned} \int_{-\infty}^{\infty} \frac{e^{-\tau^2} d\tau}{(q\tau - \Omega)^3} &= \lim_{\varepsilon \rightarrow 0} \int_{-\infty}^{\infty} \frac{e^{-\tau^2} d\tau}{(q\tau - \Omega + i\varepsilon)^3} = \\ &= -i\pi \frac{1}{2q^3} \left(e^{-\tau^2} \right)'' \Big|_{\tau=\Omega/q} + \text{V.p.} \int_{-\infty}^{\infty} \frac{e^{-\tau^2} d\tau}{(q\tau - \Omega)^3}. \end{aligned}$$

On fig. 1-3 we will present graphics of behaviour of the real part of dimensionless quantity of density of electric current

$$\text{Re } J_c(\Omega, q) = \text{V.p.} \frac{1}{8\pi\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{e^{-\tau^2} d\tau}{(q\tau - \Omega)^3}.$$

On fig. 4-6 we will present graphics of behaviour of the imaginary part of dimensionless quantity of density of electric current,

$$\text{Im } J_c(\Omega, q) = -\frac{1}{16\sqrt{\pi}q^3}(e^{-\tau^2})'' \Big|_{\tau=\Omega/q} = \frac{1}{8\sqrt{\pi}q^3}e^{-(\Omega/q)^2} \left[1 - 2\left(\frac{\Omega}{q}\right)^2 \right].$$

In case of small values of wave number from (2.6) it is received

$$\begin{aligned} j_x &= -\frac{e^3 E_y^2(x, t) N}{2m^2 \omega^3} \cdot k = -e \left(\frac{\omega_p}{\omega} \right)^2 \frac{k E_y^2}{8\pi m \omega} = . \\ &= -e \left(\frac{\Omega_p}{\Omega} \right)^2 \frac{k E_y^2}{8\pi m \omega} = -\frac{e}{8\pi} \left(\frac{\Omega_p}{\Omega} \right)^2 \frac{k E_y^2}{\Omega p_T k_T} = -\sigma_{l, tr} \frac{k E_y^2}{8\pi \Omega^3}. \end{aligned} \quad (2.8)$$

3. Quantum Maxwellian plasma

Quantum Wigner distribution function was constructed in our work [7] in general case

$$\begin{aligned} f &= f_0(P) + \frac{ev_T}{c\hbar} \mathbf{P} \mathbf{A} \frac{f_0(\mathbf{P} + \mathbf{q}/2) - f_0(\mathbf{P} - \mathbf{q}/2)}{\omega - v_T \mathbf{k} \mathbf{P}} + \frac{e^2 v_T^2 (\mathbf{P} \mathbf{A})^2}{2c^2 \hbar^2 (\omega - v_T \mathbf{k} \mathbf{P})} \times \\ &\left[\frac{f_0(\mathbf{P} + \mathbf{q}) - f_0(P)}{\omega - v_T \mathbf{k} (\mathbf{P} + \mathbf{q}/2)} + \frac{f_0(\mathbf{P} - \mathbf{q}) - f_0(P)}{\omega - v_T \mathbf{k} (\mathbf{P} - \mathbf{q}/2)} \right] - \frac{e^2 \mathbf{A}^2}{4mc^2 \hbar} \frac{f_0(\mathbf{P} + \mathbf{q}) - f_0(\mathbf{P} - \mathbf{q})}{\omega - v_T \mathbf{k} \mathbf{P}}. \end{aligned}$$

By definition, the electric current density is equal

$$\mathbf{j}(\mathbf{r}, t) = e \int \mathbf{v}(\mathbf{r}, \mathbf{p}, t) f(\mathbf{r}, \mathbf{p}, t) d^3 v. \quad (3.1)$$

Substituting in equality (4.1) obvious expression for speed

$$\mathbf{v}(\mathbf{r}, \mathbf{P}, t) = v_T \mathbf{P} - \frac{e \mathbf{A}(\mathbf{r}, t)}{mc},$$

and distribution function.

Leaving linear and square-law expressions concerning vector potential of electromagnetic field, we receive

$$\begin{aligned} \mathbf{j} &= ev_T^3 \int \left[v_T \mathbf{P} f_1 - \frac{e}{mc} \mathbf{A} f_0(P) \right] d^3 P + \\ &+ ev_T^3 \int \left[v_T \mathbf{P} f_2 - \frac{e}{mc} \mathbf{A} f_1 \right] d^3 P. \end{aligned} \quad (4.2)$$

Let us show, that the formula (4.2) for electric current density contains two non-zero components: $\mathbf{j} = (j_x, j_y, 0)$. One component j_y is linear on potential of an electromagnetic field and are directed lengthways field. It is the known formula for electric current density, so-called "transversal current". The second component j_x is quadratical on potential of field also it is directed along the wave vector. It is "longitudinal current".

The first composed in (4.2) is linear on vector potential expression, and second is square-law. We will write out these composed in obvious kind

$$\mathbf{j}^{\text{linear}} = ev_T^3 \int \left[\frac{ev_T^2}{c\hbar} \mathbf{P}(\mathbf{PA}) \frac{f_0(\mathbf{P} + \mathbf{q}/2) - f_0(\mathbf{P} - \mathbf{q}/2)}{\omega - v_T \mathbf{kP}} - \frac{e\mathbf{A}}{mc} f_0(P) \right] d^3P \quad (4.3)$$

and

$$\begin{aligned} \mathbf{j}^{\text{quadr}} = ev_T^3 \int & \left[-\frac{e^2 v_T \mathbf{A}(\mathbf{PA})}{mc^2 \hbar} [f_0(\mathbf{P} + \frac{\mathbf{q}}{2}) - f_0(\mathbf{P} - \frac{\mathbf{q}}{2})] + \right. \\ & + \frac{e^2 v_T^3 \mathbf{P}(\mathbf{PA})^2}{2c^2 \hbar^2} \left[\frac{f_0(\mathbf{P} + \mathbf{q}) - f_0(P)}{\omega - v_T \mathbf{k}(\mathbf{P} + \mathbf{q}/2)} - \frac{f_0(P) - f_0(\mathbf{P} - \mathbf{q})}{\omega - v_T \mathbf{k}(\mathbf{P} - \mathbf{q}/2)} \right] - \\ & \left. - \frac{e^2 v_T \mathbf{PA}^2}{4mc^2 \hbar} [f_0(\mathbf{P} + \mathbf{q}) - f_0(\mathbf{P} - \mathbf{q})] \right] \frac{d^3P}{\omega - v_T \mathbf{kP}}. \end{aligned} \quad (4.4)$$

Expression (4.3) is linear expression of the electric current, found, in particular, in our previous work [7]. This vector expression contains only one the component, directed along the electromagnetic fields. Really, if wave vector to direct along an axis x i.e. to take $\mathbf{k} = k(1, 0, 0)$, and potential electromagnetic fields to direct along an axis y , i.e. to take $\mathbf{A}(\mathbf{r}, t) = (0, A_y(x, t), 0)$, from the formula (4.3) we receive

$$j_y^{\text{linear}} = -\frac{e^2 v_T^3 A_y}{mcq} \int \left(\frac{f_0(P_x + q/2) - f_0(P_x - q/2)}{P_x - \Omega/q} P_y^2 + q f_0(P) \right) d^3P. \quad (4.5)$$

Here

$$f_0(P_x \pm q/2) = \frac{N}{\pi^{3/2} v_T^3} e^{-(P_x \pm q)^2 - P_y^2 - P_z^2}, \quad \Omega = \frac{\omega}{k_T v_T}.$$

Let us consider expression for an electric current (4.4), proportional to square of potential of electromagnetic field. Let us notice, that the first composed in this expression is equal to zero. Hence, this expression becomes simpler

$$\mathbf{j}^{\text{quadr}} = ev_T^3 \int \left[\frac{e^2 v_T^3 \mathbf{P}(\mathbf{P}\mathbf{A})^2}{2c^2 \hbar^2 (\omega - v_T \mathbf{k}\mathbf{P})} \left[\frac{f_0(\mathbf{P} + \mathbf{q}) - f_0(P)}{\omega - v_T \mathbf{k}(\mathbf{P} + \mathbf{q}/2)} + \frac{f_0(\mathbf{P} - \mathbf{q}) - f_0(P)}{\omega - v_T \mathbf{k}(\mathbf{P} - \mathbf{q}/2)} \right] - \frac{e^2 v_T \mathbf{P}\mathbf{A}^2}{4mc^2 \hbar} \frac{f_0(\mathbf{P} + \mathbf{q}) - f_0(\mathbf{P} - \mathbf{q})}{\omega - v_T \mathbf{k}\mathbf{P}} \right] d^3 P. \quad (4.6)$$

Let us notice, that vector expression (4.6) contains one non-zero the electric current component, directed along the wave vector

$$j_x^{\text{quadr}} = \frac{e^3 v_T^2 A_y^2}{2c^2 m^2} \int \left[\left[\frac{f_0(P_x + q) - f_0(P)}{qP_x + q^2/2 - \Omega} + \frac{f_0(P_x - q) - f_0(P)}{qP_x - q^2/2 - \Omega} \right] P_y^2 + \frac{f_0(P_x + q) - f_0(P_x - q)}{2} \right] \frac{P_x d^3 P}{qP_x - \Omega}. \quad (4.7)$$

Let us lead to the form convenient for calculations, the formula (4.7) for density of a longitudinal current.

Let us consider the first integral from (4.7). We will calculate the internal integrals in a plane (P_y, P_z) , passing to polar coordinates

$$\int f_0(P_x \pm q) P_y^2 dP_y dP_z = \frac{N e^{-(P_x \pm q)^2}}{2v_T^3 \sqrt{\pi}},$$

$$\int f_0(P_x \pm q) dP_y dP_z = \frac{N e^{-(P_x \pm q)^2}}{v_T^3 \sqrt{\pi}}.$$

Thus, size of generated longitudinal current into quantum plasma is equal

$$j_x^{\text{quant}} = \frac{e^3 N A_y^2}{4m^2 c^2 v_T \sqrt{\pi}} \int_{-\infty}^{\infty} \left[\frac{e^{-(P_x+q)^2} - e^{-P_x^2}}{qP_x + q^2/2 - \Omega} + \frac{e^{-(P_x-q)^2} - e^{-P_x^2}}{qP_x - q^2/2 - \Omega} + e^{-(P_x+q)^2} - e^{-(P_x-q)^2} \right] \frac{P_x dP_x}{qP_x - \Omega}. \quad (4.8)$$

Let us transform the expression standing in integral (4.8). At first let us pass from potential to intensity of the field $A_y = -(ic/\omega)E_y$. We will receive

$$\begin{aligned} \frac{e^3 N A_y^2}{4m^2 c^2 v_T \sqrt{\pi}} &= -\frac{e^3 N E_y^2}{4m^2 v_T \omega^2 \sqrt{\pi}} = -\frac{e \omega_p^2 E_y^2}{16\pi \sqrt{\pi} p_T \omega^2} = \\ &= -\frac{e \Omega_p^2}{16\pi \sqrt{\pi} p_T k_T} \frac{k}{\Omega^2 q} E_y^2 = -\frac{\sigma_{l,tr}}{16\pi \sqrt{\pi}} \frac{k}{\Omega^2 q} E_y^2, \end{aligned}$$

where the quantity of longitudinal-transversal conductivity $\sigma_{l,tr}$ was introduced earlier: $\sigma_{l,tr} = e \Omega_p^2 / (p_T k_T)$.

Now equality (4.8) we will present in the form

$$j_x^{\text{quant}} = J_q(\Omega, q) \sigma_{l,tr} k E_y^2 = J_q(\Omega, q) \sigma_{l,tr} \frac{\omega}{c} E_y H_z. \quad (4.9)$$

In (4.9) $J_q(\Omega, q)$ is the density of dimensionless longitudinal current,

$$J_q(\Omega, q) = -\frac{1}{16\pi \sqrt{\pi} \Omega} \int_{-\infty}^{\infty} \frac{[3q^2 + 2(q\tau - \Omega)^2 - q^4/2] e^{-\tau^2} d\tau}{[(q\tau - \Omega)^2 - q^4/4][(q\tau - \Omega)^2 - q^4]}. \quad (4.10)$$

At calculation singular integral from (4.10), which not writing out let us designate through $I(\Omega, q)$, it is necessary to take advantage known Landau rule. Then

$$I(\Omega, q) = \text{Re } I(\Omega, q) + i \text{Im } I(\Omega, q).$$

Here

$$\text{Re } I(\Omega, q) = \text{V.p.} \int_{-\infty}^{\infty} \frac{[3q^2 + 2(q\tau - \Omega)^2 - q^4/2] e^{-\tau^2} d\tau}{[(q\tau - \Omega)^2 - q^4/4][(q\tau - \Omega)^2 - q^4]},$$

symbol V.p. means a principal value of integral,

$$\begin{aligned} \text{Im } I(\Omega, q) &= -\pi \frac{\Omega}{q^2} \left\{ -\frac{4}{q^2} \left[e^{-(\Omega/q+q/2)^2} - e^{-(\Omega/q-q/2)^2} \right] + \right. \\ &\quad \left. + \left(\frac{2}{q^2} + 1 \right) \left[e^{-(\Omega/q+q)^2} - e^{-(\Omega/q-q)^2} \right] \right\}. \end{aligned}$$

Equality (4.10) for density of longitudinal current we will present in the vector form

$$\mathbf{j}^{\text{quant}} = J_q(\Omega, q)\sigma_{l,tr}\mathbf{kE}_{tr}^2,$$

or

$$\mathbf{j}^{\text{quant}} = J_q(\Omega, q)\sigma_{l,tr}\frac{\omega}{c}[\mathbf{E}, \mathbf{H}].$$

Let us show, that at small values of wave number density longitudinal current both in quantum and in classical plasma coincide.

According to (4.7) at small q after linearization $f_0(P_x \pm q)$ and transition from vector potential to electromagnetic field we receive

$$\begin{aligned} j_x^{\text{quant}} &= -\frac{e^3 N q E_y^2}{m^2 v_T \omega^2 \Omega \pi^{3/2}} \int e^{-P_x^2} P_x^2 d^3 P = -\frac{e^3 N q E_y^2}{2 m^2 v_T \Omega \omega^2} = \\ &= -\frac{e \omega_p^2 k E_y^2}{8 \pi p_T k_T \Omega \omega^2} = -\frac{e k E_y^2}{8 \pi p_T k_T \Omega} \left(\frac{\Omega_p}{\Omega} \right)^2 = -\sigma_{l,tr} \frac{k E_y^2}{8 \pi \Omega^3}, \end{aligned}$$

that in accuracy coincides with expression (2.8) for the classical plasma. These expressions we will copy in the vector form

$$\mathbf{j}^{\text{long}} = -\sigma_{l,tr} \frac{1}{8 \pi \Omega^3} \cdot \mathbf{kE}_{tr}^2 = -\sigma_{l,tr} \frac{1}{8 \pi \Omega^3} \frac{\omega}{c} [\mathbf{E}, \mathbf{H}].$$

3. Conclusions

In the present work the solution of Vlasov equation for collisionless Maxwellian plasmas is used. For the solution it the method of consecutive approximations is used.

As small parametre the quantity of the vector potential of electromagnetic field (or to it proportional quantity of intensity of electric field) is considered.

At use of approximation of the second order it appears, that the electromagnetic field generates an electric current directed along the wave vector, and proportional to the size of square of electric field.

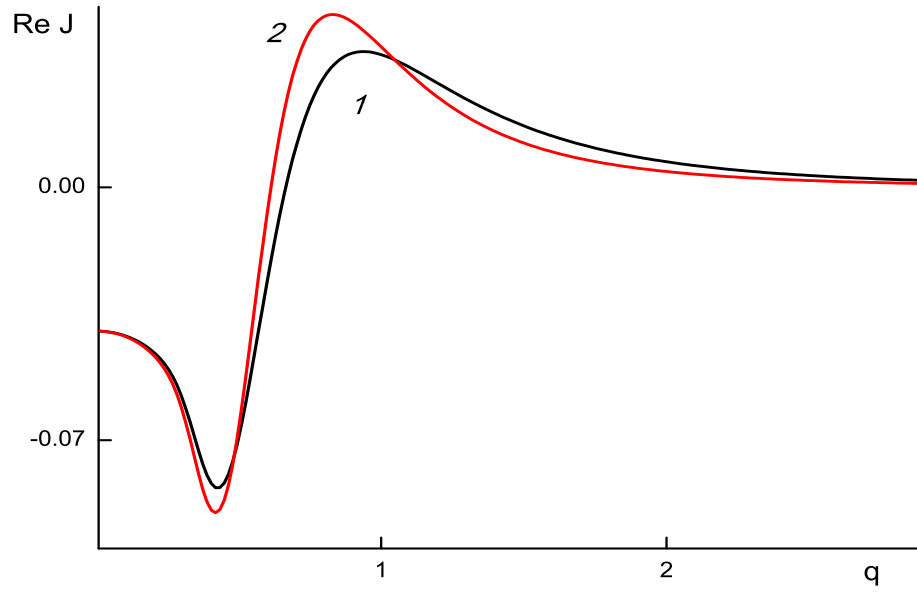


Fig. 1. Real part of longitudinal electric current density, $\Omega = 1$. Curves 1, 2 correspond to classical Maxwellian plasma and classical Fermi–Dirac plasma, dimensionless chemical potential $\alpha = 0$.

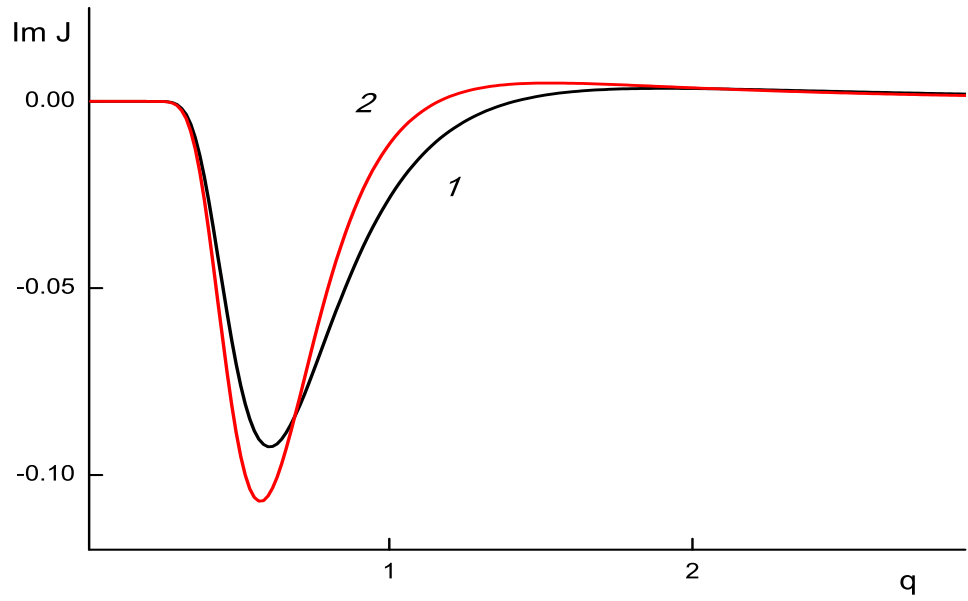


Fig. 2. Imaginary part of longitudinal electric current density, $\Omega = 1$. Curves 1, 2 correspond to classical Maxwellian plasma and classical Fermi–Dirac plasma, dimensionless chemical potential $\alpha = 0$.

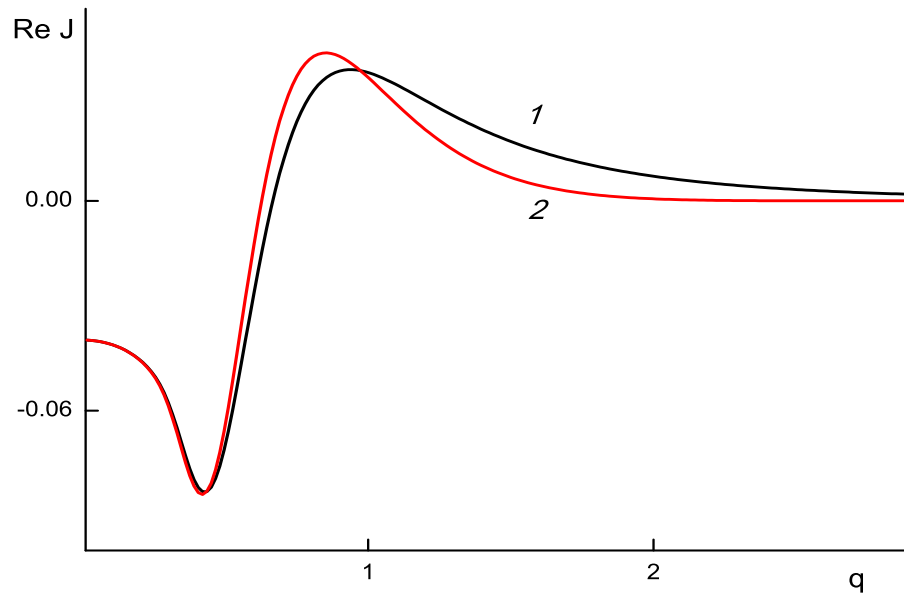


Fig. 3. Real part of longitudinal electric current density. Curves 1, 2 correspond to Maxwellian classical and quantum plasma.

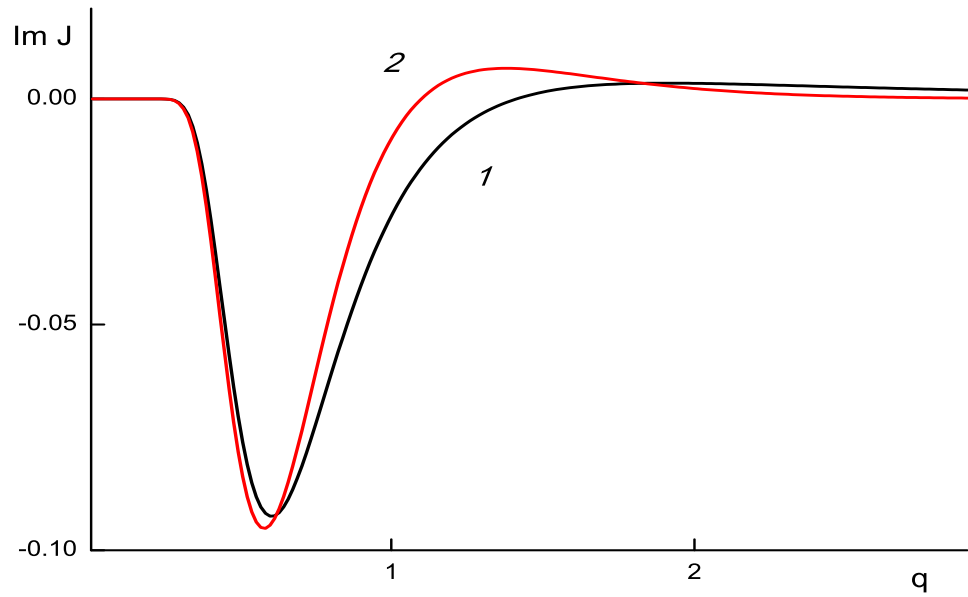


Fig. 4. Imaginary part of longitudinal electric current density. Curves 1, 2 correspond to Maxwellian classical and quantum plasma.

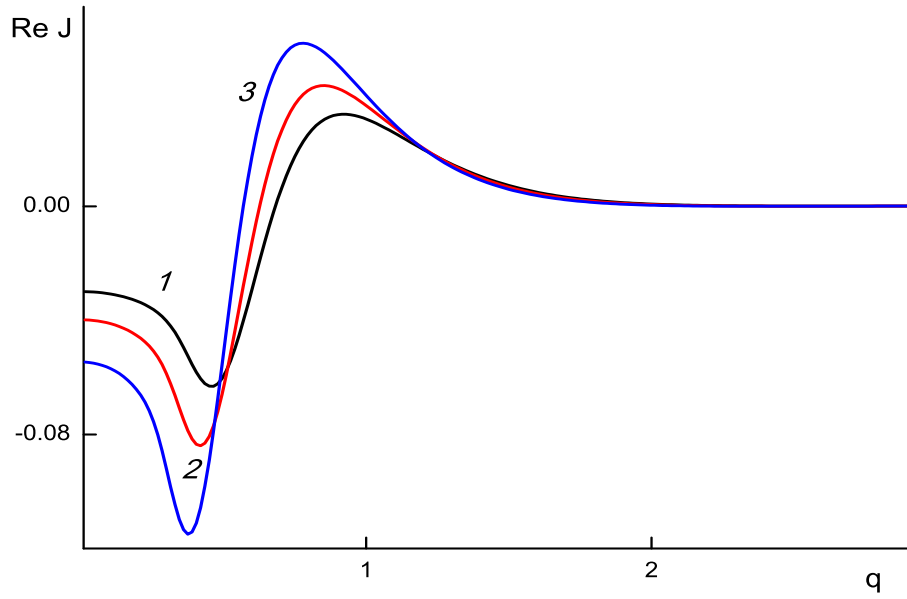


Fig. 5. Real part of longitudinal electric current density of quantum Maxwellian plasma. Curves 1, 2, 3 correspond to values of dimensionless frequency of electromagnetic field $\Omega = 1.1, 1, 0.9$.

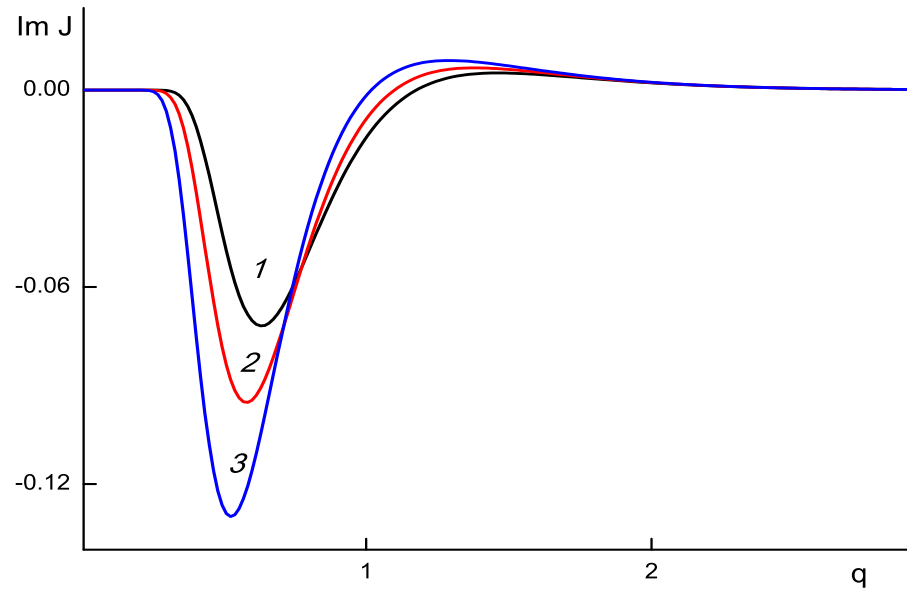


Fig. 6. Imaginary part of longitudinal electric current density, of quantum Maxwellian plasma. Curves 1, 2, 3 correspond to values of dimensionless frequency of electromagnetic field $\Omega = 1.1, 1, 0.9$.

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